



EJENS

European Journal of Engineering and Natural Sciences



e-ISSN : 2458-8156

Control Algorithms for Bosonic Quantum Batteries: Parallel versus Collective Charging

Sergey Borisenok^{1,2*}

¹Abdullah Gül University, Faculty of Engineering, Department of Electrical and Electronics Engineering, 38080, Kocasinan / Kayseri, Turkey.

²Boğaziçi University, Feza Gürsey Center for Physics and Mathematics, 34684, Üsküdar / Istanbul, Turkey.

*Corresponding Author email: sergey.borisenok@agu.edu.tr

Abstract

Very recently great progress has been achieved for a new type of quantum device: quantum batteries (QBs) which are capable to be charged, to store energy and finally to transfer it to consumption centers and other quantum devices. We develop here our theoretical feedback control algorithm for charging Bosonic quantum batteries with two different topologies of interaction between the charger and the battery set: parallel versus collective transfer of energy. The model of QB is composed of non-mutually interacting elements (quantum harmonic oscillators) in a Markovian bath. The charger of such a battery is implemented via the field which is pumping the energy into the batteries. We study the control approach in the form of Kolesnikov's target attractor to track (i.e. to drive dynamically) the charging power of the batteries. We discuss the pros and cons of the control models for different charging schemes; demonstrate their efficiency, robustness and stability. The proposed algorithms can be applied to other physical types of quantum devices: Dicke QBs and spin QBs.

Keywords

Feedback control, Quantum battery, Markovian environment

1. INTRODUCTION

Quantum Battery (QB) represented by systems with distinct quantum states must be able to perform its three basic tasks [1]:

- It can be charged, i.e. it can be transferred from its lower energy state to more energetic states. The important fact is that charging / discharging processes are not necessarily unitary, and during this process, QB may interact with its environment.
- It can efficiently store the transferred energy at the upper energy levels.
- It can transfer the stored energy to consumption centers.

The schematic diagram of an open Quantum Battery B interacting with charger A is represented in Figure 1. Each of the sub-systems, A and B , are coupled individually into an environment. In addition, one can apply an external control field to charger A [2].

There are different types of quantum batteries that could be realized in particular systems:

- Dicke QB, where one cavity mode acting as the charger is coupled to N qubits, which play the role of the battery [3, 4];
- Spin QB composed by N qubits, acting as a charger, this charger is coupled to another set of N qubits, which play the role of the battery [5, 6];
- Bosonic harmonic oscillator QB, where the sub-system B is composed of N non-mutually interacting elements [7, 8].

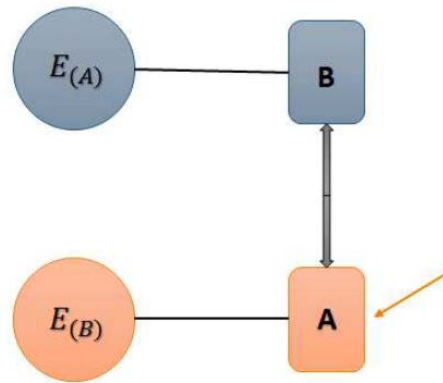


Figure 1. Diagram of an open Quantum Battery B interacting with the charger A , based on [2].

Here we discuss the last type of QB: the Bosonic quantum battery with two alternative charging schemes [9], see Figure 2.

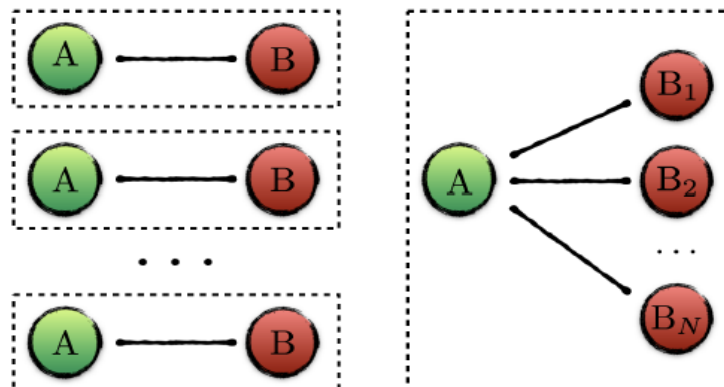


Figure 2. Two basic schemes of the parallel (left) versus collective (right) QB charging, based on [9].

The performance of QB can be enhanced by increasing the coupling strength between the nearest-neighbor environments and decreasing the size of the environments [10]. A well-engineered quantum battery is capable to store relatively more energy as compared with its non-engineered counterpart over the course of the storage phase, and the excess in such stored energy is independent of the quantum system size [11].

It has been proved also that for an array of N qubits an N -fold advantage in power per qubit can be achieved when global operations are permitted [12].

Recently, different control algorithms have been applied to quantum batteries: like the open-loop harmonic control [13] and the linear feedback [14]. Even linear feedback counteracts the randomizing influence of environmental noise and allow for stable and effective battery charging [14].

Our theoretical feedback control algorithm is applied for charging Bosonic quantum batteries with two different topologies of interaction between the charger and the battery set: parallel versus collective transfer of energy. We use Kolesnikov's target attractor algorithm to track (i.e. to drive dynamically) the charging power of the batteries.

2. MATHEMATICAL MODEL FOR BOSONIC QUANTUM BATTERY

The model of QB is composed of non-mutually interacting elements (quantum harmonic oscillators) in a Markovian bath. The charger for such a battery is implemented via the field which controls pumping the energy into the batteries.

2.1. Ergotropy and charging power

The energy storage of QB depends on the time-independent reference Hamiltonian H with the finite Hilbert space of the battery system. The useful energy exacted from QB in the state ρ and its energetically lowest accessible state σ_ρ is defined via its **ergotropy** [15]:

$$W = \text{Tr}(\rho H) - \text{Tr}(\sigma_\rho H). \quad (1)$$

The time derivative of (1) is called the **charging power** of the battery:

$$p = \frac{dW}{dt}. \quad (2)$$

We will use our control algorithm to drive the parameter (2) dynamically, i.e. we will discuss the tracking of charging power p as a given target function.

2.2. Bosonic harmonic oscillator quantum battery

The system includes the sub-system A (charger) and the sub-system B (battery) with the corresponding Hamiltonians H_A and H_B and the Hamiltonian H_1 for the coupling between A and B [1]:

$$H(t) = H_A + H_B + u(t)H_1, \quad (3)$$

where

$$\begin{aligned} H_A &= \omega_0 a^\dagger a; \\ H_B &= \omega_0 \sum_k b_k^\dagger b_k; \\ H_1 &= g \sum_k (ab_k^\dagger + a^\dagger b_k), \end{aligned} \quad (4)$$

with the corresponding creation-annihilation operators; and $u(t)$ is a time-dependent external control parameter. The bosonic harmonic oscillator QB is composed of N non-mutually interacting elements marked with the index k .

Let's consider a single-qubit based bosonic QB in the form of a quantum oscillator with the density operator ρ in a Markovian bath. Such a system is described with the Lindblad-type operator equation [16]:

$$\frac{d\rho}{dt} = -i[H_0 + u(t)\hat{Q}, \rho] + \hat{L}[\rho], \quad (5)$$

with the terms based on the creation-annihilation operators:

$$H_0 = \left(\omega_0 + \frac{1}{2} \right) b^\dagger b; \quad \hat{Q} = \frac{b^\dagger + b}{\sqrt{2\omega_0}}; \quad \hat{P} = i\sqrt{\frac{\omega_0}{2}}(b^\dagger - b). \quad (6)$$

The Lindblad operator in (5) has a form:

$$\hat{L}[\rho] = \gamma \cdot (n(t) + 1) (2b\rho b^\dagger - \rho b^\dagger b - b^\dagger b\rho) + \gamma \cdot n(t) (2b^\dagger \rho b - b b^\dagger \rho - \rho b b^\dagger). \quad (7)$$

The positive parameter γ plays here a role of coupling constant between the qubit and the environment. The external fields $u(t)$ and $n(t)$ are our control parameters.

2.3 Reduced system of differential equations

The quantum model (5)-(7) can be re-written in the form of the dynamical system with real ordinary differential equations [17]:

$$\begin{aligned}
\frac{dE}{dt} &= 2\gamma \cdot (\omega_0 n(t) - E) - u(t)P ; \\
\frac{dQ}{dt} &= P - \gamma \cdot Q ; \\
\frac{dP}{dt} &= -\omega_0^2 Q - \gamma \cdot P - u(t) ,
\end{aligned} \tag{8}$$

expressed via the ‘quasi-classical’ variables:

$$E(t) = \text{Tr}(H_0 \rho) ; Q(t) = \text{Tr}(\hat{Q} \rho) ; P(t) = \text{Tr}(\hat{P} \rho) . \tag{9}$$

The ergotropy (1) of the system (8) is defined as:

$$W(t) = E(t) - E_0 , \tag{10}$$

Where E_0 stands for the lowest accessible passive battery state.

3. CONTROL OVER THE CHARGING POWER

To perform control over the charging power we use here Kolesnikov’s Target Attractor (TA) feedback approach based on the creation in the driven dynamical system a target attractor locking the trajectories in its neighborhood [18].

3.1. Kolesnikov’s target attractor feedback

For the purpose of tracking the charging power (2) we define the goal function:

$$G(t) = p(t) - p_*(t) , \tag{11}$$

with the arbitrary differentiable target function $p_*(t)$. To form Kolesnikov’s attractor, we demand the exponentially fast convergence towards the tracking goal:

$$\frac{dG(t)}{dt} = -\frac{1}{T_1} G(t) , \tag{12}$$

with a positive time constant T_1 . Particularly, for model (8), we define the TA control equations as the following set:

$$\begin{aligned}
\frac{dE}{dt} &= -\frac{1}{T_1} (E - E_0) ; \\
\frac{dP}{dt} &= -\frac{1}{T_2} (P - P_*) .
\end{aligned} \tag{13}$$

Here T_2 is also a positive constant. The system (13) has the solutions:

$$\begin{aligned}
E(t) &= [E(0) - E_0] \cdot e^{-t/T_1} + E_0 ; \\
P(t) &= P(0) \cdot e^{-t/T_2} + P_* \cdot (1 - e^{-t/T_2}) .
\end{aligned} \tag{14}$$

The substitution of (14) into the system (8) restores the form of both control fields for a single battery element:

$$\begin{aligned}
\frac{dn(t)}{dt} &= \frac{p(t)}{\omega_0} + \frac{1}{2\gamma\omega_0} \left[P \frac{du(t)}{dt} + u(t) \frac{dP}{dt} + \frac{dp_*(t)}{dt} - \frac{1}{T_1} (p(t) - p_*(t)) \right] ; \\
u(t) &= \frac{P - P_*}{T_2} - \gamma \cdot P - \omega_0^2 Q .
\end{aligned} \tag{14}$$

By Eqs (14) we can study the achievability of the control goal.

3.2. Achievability of the control goal as $t \gg T_1, T_2$: parallel vs collective charging

Let's check the achievability of the control goal at the limit: $t \gg T_1, T_2$.

For the **parallel scheme** (see Fig.2, left) we can define different target P_* for each k -th battery element:

$$P_{*,k} = -\frac{\omega_0^2}{\gamma} Q_k(0) . \quad (15)$$

In this case, we can re-write the control fields (14) as:

$$\begin{aligned} u_k(t) &= \omega_0^2 Q_k(0) \cdot \left(1 - \frac{\omega_0^2}{\gamma}\right) \cdot (1 - e^{-\gamma t}) ; \\ n_k(t) &= \int_0^t \left\{ \frac{p_*(\tau)}{\omega_0} + \frac{1}{2\gamma\omega_0^2} \cdot \frac{dp_*(\tau)}{d\tau} - \frac{\omega_0^2}{2\gamma} \left(1 + \frac{\omega_0^2}{\gamma^2}\right) Q_k^2(0) e^{-\gamma\tau} \right\} d\tau . \end{aligned} \quad (16)$$

For the **collective scheme** (see Fig.2, right) charger A must drive each battery for the same target state: $P_k \rightarrow P_*$, $\forall k$. By that the dynamical parameter Q_k can be evaluated as:

$$Q_k \cong \frac{P_*}{\gamma} + \left[Q_k(0) - \frac{P_*}{\gamma} \right] \cdot e^{-\gamma t} . \quad (17)$$

By (8) that implies for the control field u_k :

$$u_k \cong -\frac{\omega_0^2 + \gamma^2}{\gamma} P_* - \omega_0^2 \left[Q_k(0) - \frac{P_*}{\gamma} \right] \cdot e^{-\gamma t} . \quad (18)$$

In the case, if we demand the target P_* to be zero:

$$P_* = 0 , \quad (19)$$

and the pair of control fields (14) becomes:

$$\begin{aligned} u_k &\cong -\omega_0^2 Q_k(0) \cdot e^{-\gamma t} ; \\ n_k(t) &= \int_0^t \left[\frac{p(\tau)}{\omega_0} + \frac{1}{2\gamma\omega_0} \frac{dp_*(\tau)}{d\tau} \right] d\tau . \end{aligned} \quad (20)$$

Now we can compare the shapes of the control fields (14) for the parallel and collective approaches. For the parallel case (16), the control fields u_k are closed to 0 for each k -th battery: $u_k \cong 0$, while the control fields n_k are more complex for the practical computations. For the collective case of charging (20), the fields u_k tend to 0 asymptotically: $u_k \cong -\omega_0^2 Q_k(0) \cdot e^{-\gamma t}$, while n_k are more simple for the numerical computations.

4. CONCLUSIONS

The algorithm of the control over the charging power proposed here has the following basic characteristics. This algorithm is **universal** and does not depend on the initial conditions of the dynamical variables. It is **robust** and stable under the perturbation of the initial conditions and a relatively small external noise.

The proposed algorithm **can be extended for different physical realizations of quantum batteries**: Dicke QB, spin QB; and for all working stages of the QB (charging, long time storage and the energy transfer to a consumption center or engine) and it can be easily extended for a multi-qubit model.

From the point of practical computation of the coherent and non-coherent fields u and n the control algorithm for Bosonic quantum batteries demonstrates different features for two alternative schemes of charging:

- For the **parallel scheme** of charging the control signal u (interaction between the battery and the charger) tends to 0 as the time is increasing. From another hand, the control signal n (interaction with the Markovian environment) is more complex for computation.

- For the **collective scheme** of charging the control signal u (interaction between the battery and the charger) tends exponentially to 0 as the time is increasing. At the same time, the control signal n (interaction with the Markovian environment) becomes simpler for the numerical computation.

CONFLICT OF INTEREST STATEMENT

The author declares that there is no conflict of interest.

REFERENCES

- [1]. D. Ferraro, M. Campisi, G. M. Andolina, V. Pellegrini, M. Polini, "High-power collective charging of a solid-state quantum battery", *Phys. Rev. Lett.*, vol. 120(11), p. 117702, 2018.
- [2]. F. H. Kamin, F. T. Tabesh, S. Salimi, F. Kheirandish, A. C. Santos, "Non-Markovian effects on charging and self-discharging processes of quantum batteries", *N. J. Phys.*, vol. 22, p. 083007, 2020.
- [3]. X. Zhang, M. Blaauboer, "Enhanced energy transfer in a Dicke quantum battery", arXiv:1812.10139v1, 2018.
- [4]. S. Borisenok, "Control over cavity assisted charging for Dicke quantum battery", *European Int. J. of Sci. and Technology*, vol. 9, no. 6, pp. 1-7, 2020.
- [5]. S. Ghosh, A. Sen(De), "Dimensional enhancements in a quantum battery with imperfections", *Phys. Rev. A*, vol. 105, p. 022628, 2022.
- [6]. F. Zhao, F.-Q. Dou, Q. Zhao, "Charging performance of the Su-Schrieffer-Heeger quantum battery", *Phys. Rev. Research*, vol. 4, p. 013172, 2022.
- [7]. A. I. Arbab, "The complex quantum harmonic oscillator model", *Europhysics Letters*, vol. 9843, no.3, p. 30008, 2012.
- [8]. T. K. Konar, L. G. Ch. Lakkaraju, S. Ghosh, A. Sen De, "Quantum battery with ultracold atoms: Bosons vs. Fermions", arXiv:2109.06816v1, 2021.
- [9]. G. M. Andolina, M. Keck, A. Mări, V. Giovannetti, M. Polini, "Quantum versus classical many-body batteries", *Phys. Rev. B*, vol. 99, p. 205437, 2019.
- [10]. K. Xu, H.-J. Zhu, G.-F. Zhang, W.-M. Liu, "Enhancing the performance of an open quantum battery via environment engineering", *Phys. Rev. E*, vol. 104, p. 064143, 2021.
- [11]. J. Liu, D. Segal, "Boosting quantum battery performance by structure engineering", arXiv:2104.06522v1, 2021.
- [12]. F. Binder, S. Vinjanampathy, K. Modi, J. Goold, "Quantacell: Powerful charging of quantum batteries", *New Journal of Physics*, vol. 17, p. 075015, 2015.
- [13]. S. Mondal, S. Bhattacharjee, "Charging of quantum battery with periodic driving", arXiv:2112.10451v1, 2021.
- [14]. M. T. Mitchison, J. Goold, J. Prior, "Charging a quantum battery with linear feedback control", *Quantum*, vol. 5, p. 500, 2021.
- [15]. G. Francica, J. Goold, F. Plastina, M. Paternostro, "Daemonic ergotropy: enhanced work extraction from quantum correlations", *npj Quantum Inf.*, vol. 3, p. 12, 2017.
- [16]. A. Pechen, "Engineering arbitrary pure and mixed quantum states", *Phys. Rev. A*, vol. 84, p. 042106, 2011.
- [17]. S. Borisenok, "Ergotropy of quantum battery controlled via target attractor feedback", *IOSR Journal of Applied Physics*, vol. 12(3), pp.43-47, 2020.
- [18]. A. Kolesnikov, *Synergetic control methods of complex systems*, Moscow: URSS Publ., 2012.